RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2019

FIRST YEAR [BATCH 2018-21]

PHYSICS (Honours)

Paper : II

Date : 16/05/2019

Time : 11 am – 3 pm

Full Marks : 100

Group - A

(*Answer <u>any three</u> questions*) [3×10]

1. a) Show that the points of any plane passing through the origin of three-dimensional Euclidian space form a vector space. [4]

   b) Show that set S of the following four matrices

   $|e_1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, |e_2\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, |e_3\rangle = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, |e_4\rangle = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is a basis for the vector space of 2×2 matrices. What is the dimension of this vector space. [2+1]

   c) What do you mean by an inverse operator? Can we have an inverse operator for any linear operator? Explain. [2]

   d) Show that if $\hat{A}$ is linear , then $\hat{A}^{-1}$ is also linear. [1]

2. a) Construct an orthonormal set of eigen vectors for the matrix A. Also find the eigenvalues.

   $$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

   Hence diagonalise the matrix. [3+2+2]

   b) Show that trace of a matrix remains invariant under similarity transformation. [1]

   c) Show that the sum of the eigenvalues of a matrix is equal to the trace of that matrix. [2]

3. a) Prove the vector identity using tensor rotation.

   $\vec{\nabla}\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} - \vec{B}(\vec{\nabla}\cdot\vec{A}) + \vec{A}(\vec{\nabla}\cdot\vec{B}) - (\vec{A}\cdot\vec{\nabla})\vec{B}$ [4]

   b) Show that, $\Gamma(n) = \int_0^1 \left(\log\frac{1}{y}\right)^{n-1} dy \;, n > 0$ [3]

   c) Express in terms of gamma function

   $I = \int_0^1 \frac{dx}{\sqrt{(1-x^4)}}$ [3]

4. a) Prove that following recursion relation for the Legendre polynomials

   i) $lP_l(x) = (2l-1)xP_{l-1}(x) - (l-1)P_{l-2}(x)$

   ii) $lP_l(x) = xP_l'(x) - P_{l-1}'(x)$

   Hint : Generating function for the Legender polynomial is

   $\phi(x,h) = (1-2xh+h^2)^{-\frac{1}{2}} = \sum_{l=0}^{\infty} h^l P_l(x), \; for \; |h| < 1$ [4]

   b) Show that the othonormality for the Legendre polynomial is

$$\int_{-1}^{+1} P_l(x) P_m(x)\, dx = \frac{2}{2l+1}\delta_{lm}$$ [6]

5. a) Solve the wave equation in two-dimensional polar coordinates, i.e. [5]

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

b) A circular membrane is attached to a rigid frame at r = a , and someone is beating the membrane at the centre so that the following initial conditions hold

$\psi(r=0, t=0) = d$, and $\frac{\partial\psi}{\partial t}\Big|_{t=0} = 0.$

Determine the frequency and amplitude of vibration. [5]

Group - B

(*Answer <u>any two</u> questions*) [2×10]

6. a) Three particles of equal mass A, B and C are moving on a straight line with velocities $4v_0$, $2v_0$ and v_0 , respectively. Initially they are located at -1, 0, +1 along the line. Assuming all collision to be elastic, find the final velocities of all the particles. [2]

b) Show that the kinetic energy T, and angular momentum $\vec{L}$ of a two-particle system are given by $T = \frac{1}{2}M\vec{V}_c^2 + \frac{1}{2}\mu\vec{v}^2$ and $\vec{L} = M\vec{R}\times\vec{V}_c + \mu\vec{r}\times\vec{v}$, where $M = m_1 + m_2$, $\vec{V}_c =$ velocity of centre of mass, μ = reduced mass, $\vec{v} =$ relative velocity. [4]

c) Show that in a one dimensional elastic collision the speed of the centre of mass of two particles, m_1 having initial velocity u_1, and m_2 moving with initial velocity u_2 is given by $V = \left(\frac{m_1}{m_1+m_2}\right)u_1 + \left(\frac{m_2}{m_1+m_2}\right)u_2$ [4]

7. a) A non-inertial reference frame R undergoes both translational and rotational motion relative to an inertial system, S. A moving particle P has position vector $\vec{r}$ relative to the moving frame, and $\vec{r}_0$ relative to the inertial frame, at any instant. If $\vec{\omega}$ is the uniform angular velocity of R w.r.t. S, show that the velocity $\vec{v}$ of particle as observed in S is given by $\vec{v} = \vec{V} + \dot{\vec{r}} + \vec{\omega}\times\vec{r}$, where $\vec{V}$ is the linear velocity of R, and the dot $(\dot{})$ denotes time derivative in the rotating system R. [4]

b) Use the result of (a) to show that the acceleration of P in S, can be expressed as, $\vec{a} = \frac{d\vec{v}}{dt} = \vec{A} + \ddot{\vec{r}} + 2\vec{\omega}\times\dot{\vec{r}} + \vec{\omega}\times(\vec{\omega}\times\vec{r})$, where $\vec{A} = \frac{d\vec{V}}{dt}$. Interpret the various terms on the RHS. [3]

c) What is coriolis force. Discuss the effect of coriolis force produced as a result of earth's rotation. [3]

8. a) Set up a suitable coordinate system in the rotating frame of the earth, at any point O on the surface in the Northern hemisphere, at latitude λ. Express the angular velocity $\vec{\omega}$ in terms of the coordinate system. [2+1]

b) Given that the equation of motion of a projectile near the surface of the earth is $\ddot{\vec{r}} = \vec{g} - 2\vec{\omega}\times\dot{\vec{r}}$ obtain a solution of this equation, vectorially or otherwise, with initial conditions : $\vec{r}(t=0) = \vec{r}_0$, $\dot{\vec{r}}(t=0) = \vec{u}_0$. Neglect all ω^2 dependent terms. [4]

c) For a particle dropped from rest from a vertical tower of height H, use (b) to show that when the particle reaches the ground, it is displaced eastward from the foot of the tower, by

$$x_0 = \frac{1}{3}\omega g\cos\lambda\left(\frac{2H}{g}\right)^{3/2}$$ [3]

9. a) State the expression for the moment of inertia of a uniform cylinder of length l and radius R about an axis through its centre and normal to its length. If the above moment of inertia is to be minimum, determine in ratio $\frac{l}{R}$ (mass of the cylinder kept constant) and show that the ratio is $\sqrt{3}:\sqrt{2}$ [4]

b) Calculate the kinetic energy, T, of a rigid body turning about a fixed point with angular velocity $\vec{\omega}$, in terms of the inertia tensor at that point. [3]

c) If $\vec{L}$ is the angular momentum of the body about the fixed point, show that the components of $\vec{L}$ satisfy the relations; [3]

$$L_i = \frac{\partial T}{\partial \omega_i} \quad (i = x, y, z)$$

Group - C

(*Answer <u>any two</u> questions*) [2×10]

10. a) Starting from Lorentz transformation equations derive the equations for relativistic addition of velocities. From it show that the velocity of light is an absolute constant. [5+1]

b) Deduce the minimum energy of a gamma ray photon (in MeV) which can cause electron postion pair production. $\left(m_e = 9.1\times10^{-31} kg\right)$ [4]

11. An inertial frame S_2 is moving with velocity v along the +ve x-axis with respect to another inertial frame S_1. Two equal masses of same speed, measured in the S_2 frame, are moving opposite direction cause to make an elastic collision. Calculate the ratio of the two masses as measured in S_1 frame. [10]

12. a) Explain 'length contraction' and 'time dilation' in Minkowski's geometrical representation. [5]

b) The velocity of a particle is $6\hat{i} + 5\hat{j} + 4\hat{k}$ in a frame of reference $S^{/}$, moving with velocity 0.8 c along the x-axis, relative to a reference frame S at rest. What is the velocity of the particle in the later frame. [5]

13. a) Derive the relation for longitudinal Doppler effect. [4]

b) Show that $X^2 + Y^2 + Z^2 - c^2T^2$ is an invariant under Lorentz transformation. [3]

c) Calculate the velocity of a 1 MeV electron. [3]

Group - D

(*Answer <u>any three</u> questions*) [3×10]

14. a) Explain what is meant by chromatic aberration. Deduce the condition for achromatism of two lenses separated by a distance. [2+4]

b) Two thin lenses of focal lengths f_1 and f_2 separated by a distance d have an equivalent focal length 50 cm. The combination satisfies the condition for no chromatic aberration and minimum spherical aberration. Find the values of f_1, f_2 and d. Assume that both the lenses are of the same material. [4]

15. Explain with a neat diagram the construction and working of Huygens eyepiece. Why cannot a cross-wire be used with it? [5+3+2]

16. a) State Huygens's principle. [2]

b) Describe the method of determination of wavelength of a monochromatic light by means of Lloyd's single mirror. Explain why the central fringe is black in Lloyd's mirror experiment. [4+1]

c) In the Young's double slit experiment, a thin mica sheet (n = 1.5) is introduced in the path of one of the beams. If the central fringe gets shifted by 0.2 cm, calculate the thickness of the sheet. Assume d = 0.1 cm and D = 50 cm. [3]

17. a) Illustrate how interference is generated by division of ampltitude. [2]

b) In a Newton's ring experiment, the diameters of n^{th} and $(n+10)^{th}$ bright rings are 2.18 mm and 4.51mm respectively. Calculate the refractive index of the liquid. Radius of curvature of the lens is 90 cm, and wavelength of light employed is 589.3 nm. [5]

c) Discuss the working principle of Michelson's interferometer with schematic diagram. [3]

18. a) What is multiple beam interferometry? What is the advantage of this? [2]

b) Show analytically that multiple beam interferometry produces sharper pattern compared to division of amplitude. [4]

c) Calculate the angular separation between the two components of sodium D-lines (having wavelengths, 589 nm and 589.6 nm) in second order. The number of lines per cm of the grating is 800. [4]

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